King's Rock

As a preface: I am sure there are a good chunk of people familiar with college-level probability theory, and many many more who are not, but some may be interested in learning about it. With this in mind, I tried to write this in a way that would be digestible to the common user, with some level of rigor and use of technical terminology for the dual purpose of clarity for those with a background in mathematics, and to introduce those interested in exploring the topic more deeply. Honestly I'm not sure why I spent as much time on this as I did, but it was honestly quite fun so I guess I enjoyed the refresher. Also, I'm sure there are many people far more knowledgeable in these topics than I - if you have something to add or correct, I would love to hear your feedback. **The purpose of this analysis to provide a basic quantitative analysis on King's Rocks performance as a damage multiplier.**

Let us define the **random variable** (r.v.) X as the total damage dealt by a 5-hit multi-hit move with King's Rock.

The probability distribution of X is called a **geometric distribution**. Essentially, think of each time you attack as a weighted coin-flip, on whether you flinch or not. The total amount of damage you do is governed by how many successive attacks (these would be formally called **independent Bernoulli trials**) you can land until you do not flinch. The standard mathematical formulation of this is a little counter-intuitive to the way one thinks about "success" and "failure" with king's rock, so I'll lay it out explicitly as follows.

Let $p := 0.9^5$ denote the chance of failure to flinch, and let $q := 1 - p$, *i.e.* the chance to flinch. Also, suppose there are no damage rolls and the total damage of a single attack is 1. This means 2 successive attacks has total damage 2, and so on. Essentially, our random variable X will always take a value that is a positive integer, which we will call k ($k \in \mathbb{Z}^+$). Then, the **probability mass function** (p.m.f) of X is given as

$$\mathbb{P}(X = k) = q^{k-1}p.$$

Let's give an example. Suppose we want to know the chance of flinching two times before not flinching on the third attempt, giving us a total damage output of 3. Then, the probability would be the chance of flinching twice, q^2, times the probability of not flinching the third time, p. So then, $\mathbb{P}(X = 3) = q^2 p$ as expected.

Now, we are interested in the **expectation** and **variance** of X to see how effective King's Rock is as a damage booster. This is a calculation we can generalize for all geometric distributions and simply reference, but I'll derive the expectation because I find it interesting. I assume most readers know the basics of what these terms refer to, but please Google them if you do not. Feel free to skip to the next page if you do not wish to read this. We have

$$\mathbb{E}[x] := \sum_{k=1}^{\infty} k\mathbb{P}(X = k) = \sum_{k=1}^{\infty} kq^{k-1}p.$$

This can be solved by realizing that the sum we are interested in can be rewritten as a sum of multiple geometric series. The idea is to seperate each term with coefficient k into k terms with coefficient 1, and rearrange them to sum them in a clever way. One way I like to think of this is to separate each term of the summation and lay them out in a right triangle. For example, in the top row, I would have $p, qp, q^2p, q^3p, \ldots$. In the second row, I would start below the qp term, and would write $qp, q^2p, q^3p, \ldots$. In the third row I start below the third term, and so on. For simplicity, we can also factor out the common p term. The sum of these rows are **geometric power series.** Then, the first row would sum to $1/(1-q)$, the second row to $q/(1-q)$, the third to $q^2/(1-q)$, and so on. Then, the sum of these series is also a geometric series, with sum

$$\frac{1/(1-q)}{(1-q)} = \frac{1}{p^2},$$

and of course we factored out an earlier term of p so this would become $1/p$. We can calculate the variance using similar techniques or methods of **moment generating functions,** but we will simply take the existing result that

$$\mathrm{Var}[x] = \frac{1-p}{p^2}.$$

What does this all mean in the case of King's rock? We have $\mathbb{E}[X] = 1/p = 1/0.9^5 \approx \mathbf{1.694}$, and $\mathrm{Var}[x] = (1-p)/p^2 = (1-0.9^5)/0.9^{10} \approx \mathbf{1.174}$. For a point of reference we can compare this to an item like choice band, which comes with significant drawbacks and has EV 1.5 and Variance 0. It is clear that King's Rock applied to multihit attacks can be very RNG-based due to the large variance, but has more damage output overall without the drawbacks of most damage-boosting items. Obviously this can be more nuanced due to speed tiers, priority, inner focus, etc. but at face value, this is extremely powerful.

There's one key analytical problem with this: how often do you use a King's Rock attack and flinch a single Pokemon multiple times before it faints? If the opposing Pokemon faints, there is no chance to flinch and the distribution resets. Usually, you will not attack more than 3 times before KO'ing the opponent. Let us modify our r.v. with this in mind. Let X' be the r.v. when we only attack 2 times at most, and X'' when we attack 3 times at most. Then,

$$\mathbb{P}(X' = 1) = p,\ \mathbb{P}(X' = 2) = q,$$

$$\mathbb{P}(X'' = 1) = p,\ \mathbb{P}(X'' = 2) = qp,\ \mathbb{P}(X'' = 3) = q^2.$$

Using these, and the simple formula for variance, $\text{Var}[x] = E[x^2] - E[x]^2$, we find for X' :

$$\mathbb{E}[X'] = p + 2q = 0.9^5 + 2(1 - 0.9^5) = 1.40951,$$

$$\text{Var}[X'] = p + 4q - \mathbb{E}[X']^2 = 0.9^5 + 4(1 - 0.9^5) - (1.40951) = 2.45706,$$

and for X'' :

$$\mathbb{E}[X''] = p + 2qp + 3q^2 = 0.9^5 + 2(1 - 0.9^5)0.9^5 + 3(1 - 0.9^5)^2 \approx 1.577,$$

$$\text{Var}[X''] = p + 4qp + 9q^2 - \mathbb{E}[X'']^2 = 0.9^5 + 4(1 - 0.9^5) - (1.40951) = 1.4898137604.$$

We see that in the limited case, the power of King's Rock is actually much weaker, because we cannot take full advantage of our opportunity to flinch. In the case of X' we are strictly worse than a choice band in terms of raw damage output, and for X'' we are arguably worse due to high variance, despite slightly greater EV. This is not to say that King's Rock and

Choice Band can be directly compared (they cannot), but it is a useful way to quantify the value of King's Rock as a damage booster. A deeper analysis might involve using the **central limit theorem** to look at the distribution of average damage over a large sample. I will not delve into this, but it is certainly interesting and I would recommend looking into it if you are interested.